

Intervals () end excluded · [] end included

Periods sin, cos, sec, cosec 2π · tan, cot π · {x} 1

Principal ranges $\sin^{-1}$: $[-\frac{\pi}{2}, \frac{\pi}{2}]$ $\cos^{-1}$: $[0, \pi]$ $\tan^{-1}$: $(-\frac{\pi}{2}, \frac{\pi}{2})$

1 Relations: Definition, Domain and Range

$$n(A \times B) = pq \text{ when } n(A) = p, n(B) = q$$

$$\text{relations from } A \text{ to } B = 2^{pq}$$

2 Reflexive, Symmetric and Transitive Relations

$$\text{reflexive: } 2^{n^2-n}$$

$$\text{symmetric: } 2^{(n^2+n)/2}$$

$$\text{reflexive and symmetric: } 2^{(n^2-n)/2}$$

3 Equivalence Relations and Bell Numbers

$$B_{n+1} = \sum_{k=0}^n {}^nC_k B_k, B_0 = 1$$

$$B_1 = 1, B_2 = 2, B_3 = 5, B_4 = 15$$

4 Functions: Definition, Intervals and Domain

$$(a, b)^c = (-\infty, a] \cup [b, \infty)$$

$$[a, b]^c = (-\infty, a) \cup (b, \infty)$$

- Function: each input exactly one output; vertical line test.

5 Algebra of Functions and Composite Functions

- $D(f \pm g) = D_f \cap D_g$; f/g also drops $g = 0$; $D(gof) = \{x \in D_f : f(x) \in D_g\}$.

6 One-One, Many-One, Onto and Into Functions

- One-one: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$; onto: range = codomain.

7 Bijections, Counting Functions and Composites

$$\text{functions } A \rightarrow B: m^n$$

$$\text{one-one: } {}^mP_n \ (m \geq n)$$

$$\text{onto: } \sum_{r=0}^m (-1)^r {}^mC_r (m-r)^n$$

$$\text{bijections } (m = n): n!$$

8 Inverse Functions

$$f \circ f^{-1} = I_B, f^{-1} \circ f = I_A$$

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

- f^{-1} exists $\Leftrightarrow$ bijective; graph: mirror in $y = x$.

9 Inequalities and the Wavy Curve Method

$$x^2 \leq a^2 \Leftrightarrow -a \leq x \leq a$$

$$x^2 \geq a^2 \Leftrightarrow x \leq -a \text{ or } x \geq a$$

$$ab > 0: \frac{1}{x} \in [\frac{1}{b}, \frac{1}{a}]$$

$$a < 0 < b: \frac{1}{x} \in (-\infty, \frac{1}{a}] \cup [\frac{1}{b}, \infty)$$

10 Domain and Range of Algebraic Functions

$$f(x) = a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{b^2}{4a^2} \right]$$

$$a > 0: \text{range } [-\frac{b^2}{4a}, \infty)$$

$$a < 0: \text{range } (-\infty, -\frac{b^2}{4a}]$$

11 Modulus Function

$$|x| = x \ (x \geq 0), -x \ (x < 0)$$

$$|f(x)| = f(x) \ (f \geq 0), -f(x) \ (f < 0)$$

$$|ab| = |a| |b|$$

$$|\frac{a}{b}| = \frac{|a|}{|b|}$$

$$x^2 = |x|^2 = |x^2|$$

$$|x| = a \ (a > 0) \Rightarrow x = \pm a$$

$$|a \pm b|^2 = a^2 + b^2 \pm 2ab$$

$$(|a| \pm |b|)^2 = a^2 + b^2 \pm 2|a||b|$$

12 Trigonometric Functions: Graphs and Ranges

$$\sin x \geq 0 \Leftrightarrow x \in \bigcup [2n\pi, (2n+1)\pi]$$

$$\cos x \geq 0 \Leftrightarrow x \in \bigcup [2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}]$$

$$a \cos x + b \sin x = \sqrt{a^2 + b^2} \sin(x + \alpha)$$

$$\text{range } [-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$$

13 Domain and Range of Trigonometric Expressions

- $\sqrt{1 + \tan^2 x} = |\sec x|$. $\sin x = x/10$: 7 roots.

14 Inverse Trigonometric Functions

$$\sin^{-1}(-x) = -\sin^{-1} x$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$

$$\tan^{-1}(-x) = -\tan^{-1} x$$

$$\cot^{-1}(-x) = \pi - \cot^{-1} x$$

$$\sec^{-1}(-x) = \pi - \sec^{-1} x$$

$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$$

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, |x| \leq 1$$

$$\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}, |x| \geq 1$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, x \in \mathbb{R}$$

$$\sin^{-1} \frac{1}{x} = \operatorname{cosec}^{-1} x, \cos^{-1} \frac{1}{x} = \sec^{-1} x \ (|x| \geq 1)$$

$$\tan^{-1} \frac{1}{x} = \cot^{-1} x \ (x > 0), -\pi + \cot^{-1} x \ (x < 0)$$

15 Exponential and Logarithmic Functions

- Base > 1 keeps, base in $(0, 1)$ flips; variable base $> 0, \neq 1$.

16 Greatest Integer, Fractional Part, Signum and Max-Min

$$x - 1 < [x] \leq x$$

$$[x] \geq n \Leftrightarrow x \geq n$$

$$[x] < n \Leftrightarrow x < n$$

$$\operatorname{sgn} x = 1, 0, -1 \text{ for } x > 0, x = 0, x < 0$$

$$\operatorname{sgn} x = \frac{|x|}{x}, x \neq 0$$

$$\max(a, b) = \frac{a+b+|a-b|}{2}$$

$$\min(a, b) = \frac{a+b-|a-b|}{2}$$

17 Even and Odd Functions

$$f(x) = \frac{f(x)+f(-x)}{2} + \frac{f(x)-f(-x)}{2}$$

- $E \times E = O \times O = E$; $E \times O = O$. Domain must be symmetric.

18 Periodic Functions

$$\text{period of } f(ax + b) = \frac{T}{|a|}$$

- Sums: LCM, then test smaller ($|\sin x| + |\cos x|$: $\pi/2$).

19 Functional Equations

- Additive $\rightarrow kx$; $f(xy) = f + f \rightarrow k \log x$; $f(x + y) = f \cdot f \rightarrow a^x$.

20 Transformation of Graphs

- Outside: as written; inside: reversed ($f(x + k)$ left).